

Symmetric and Asymmetric Nuclear Matter in the Thomas-Fermi Model at Finite Temperatures*

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The properties of warm symmetric and asymmetric nuclear matter are investigated in the frame of the Thomas-Fermi approximation using a recent modern parameterization of the effective nucleon-nucleon interaction of Myers and Świątecki. Special attention is paid to the liquid-gas phase transition, which is of special interest in modern nuclear physics. We have determined the critical temperature, critical density and the so-called flash temperature. Furthermore, the equation of state for cold neutron star matter is calculated.

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I. Introduction

An essential but also a very complex and complicated problem of modern physics is the structure of matter under extreme conditions of temperature and/or density. The equation of state of hot dense matter is a key ingredient for many branches of modern physics. To mention in this respect are, for instance, the understanding of the physics of the early universe, supernova explosions and protoneutron stars. A further important example are high-energy heavy-ion collisions, which provide, at present, the only laboratory means to study the behaviour of hot and dense matter. In the literature one can roughly distinguish between two theoretical attempts to investigate the equation of state, namely the non-relativistic and relativistic approaches. The majority of the relativistic treatments is performed in the framework of the relativistic Hartree approximation. Only a few investigations use the relativistic Hartree-Fock or relativistic Brückner-Hartree-Fock approximation (see, for instance, [1–3]). Most of the theoretical treatments utilize the non-relativistic scheme, using either effective density dependent interactions or the Brückner approach [4–6].

In this investigation we extend a recent modern Thomas-Fermi approach of Myers and Świątecki [7–10] to the equation of state at finite temperatures (see also [11]). Furthermore we briefly discuss the question of neutron star matter and the equation of state for higher asymmetries.

The paper is organized as follows: First, we recapitulate for the sake of completeness in Section II the basic structure of the Thomas-Fermi model [12, 13] for density and momentum dependent interactions. The extension of the model to finite temperatures is given in Section III. Section IV deals with the results of the model, and the last Section is a summary of the results with conclusions.

II. Thomas-Fermi model at $T=0$

We shall not recapitulate the basic assumptions and limitations of the Thomas-Fermi model in nuclear physics, since they are described in greater detail in the pioneering work of Myers and Świątecki [14]. In the modern version they use an effective interaction of the structure (τ denotes the isospin)

$$v_{12\tau} = -\frac{2 T_{F,\tau}}{\rho_0} g\left(\frac{r_{12}}{a}\right) \times \left(\frac{1}{2} (1 \mp \xi) \alpha - \frac{1}{2} (1 \mp \zeta) \times \left(\beta \left(\frac{p_{12}}{p_F} \right)^2 - \gamma \frac{p_F}{|p_{12}|} + \sigma \left(\frac{2\bar{\rho}}{\rho_0} \right)^{2/3} \right) \right) \quad (\text{II.1})$$

with the definition

$$\bar{\rho}^{2/3} = \frac{1}{2} (\rho_1^{2/3} + \rho_2^{2/3}), \quad (\text{II.2})$$

where ρ_1, ρ_2 denote the (proton or neutron) density at the locations \mathbf{r}_1 and \mathbf{r}_2 , respectively. The density at equi-

* Dedicated to Georg Süßmann on the occasion of his seventieth birthday.

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librium is $\rho_0 = (4 \pi/3 r_0^3)^{-1}$, and the Fermi momentum and the Fermi kinetic energies are given by $[\bar{m} := 1/2 (m_n + m_p)]$

$$p_F = \hbar \left(\frac{3}{2} \pi^2 \rho_0 \right)^{1/3}, \quad (\text{II.3})$$

$$T_F = \frac{p_F^2}{2 \bar{m}}, \quad (\text{II.4})$$

$$T_{F,\tau} = \frac{p_F^2}{2 m_\tau}. \quad (\text{II.5})$$

By means of the parameters ξ and ζ the interaction distinguishes the forces between nucleons of the same kind (upper sign) and of different kind (lower sign). With the parameter α one adjusts mainly the binding properties of nuclear matter. The repulsion is described by the momentum dependent term $\propto \beta p_{12}^2$. Since this repulsion turns out to be too strong for higher relative momenta, one corrects this deficit by the term $\propto \gamma |p_{12}|^{-1}$. In order to obtain better values for higher asymmetries, we have chosen $\xi \neq \zeta$ in contrast to former structures of Seyler and Blanchard [15]. The term proportional to $\sigma (2 \bar{\rho}/\rho_0)^{2/3}$ takes care for a better agreement with the nuclear optical potential (for more details, see [7–11]). For our task the model contains six free parameters (in the expression for the energy the parameters β and σ enter only in the combination $B = \beta + 5/6 \sigma$). They are given in Table 1. For the space dependence the standard Yukawa form is employed, i.e.

$$g \left(\frac{r_{12}}{a} \right) = \frac{1}{4 \pi a^3} \frac{\exp(-r_{12}/a)}{r_{12}/a}. \quad (\text{II.6})$$

The energy per nucleon in the Thomas-Fermi model is given by

$$u = \frac{E}{N} = \frac{1}{\rho} \frac{2}{(2 \pi \hbar)^3} \sum_{\tau} \int d^3 p_1 \left(\frac{p_1^2}{2 m_\tau} + \frac{1}{2} V_\tau(p_1) \right) \quad (\text{II.7})$$

Table 1. Parametrization of the interaction [7].

Parameter x	Value	Value x_l	Value x_u
a	0.59542 fm	—	—
α	3.60928	1.01054	2.59874
β	0.37597	0.07561	0.30036
γ	0.21329	0.04289	0.17040
σ	1.33677	0.26884	1.06793
B	1.48995	0.29964	1.19030
ξ	0.44003	—	—
ζ	0.59778	—	—

with the single-particle potential

$$V_\tau(p_1) = - \frac{2}{(2 \pi \hbar)^3} \left(\int d^3 p_2 v_{12 \tau} + \int_{-\tau} d^3 p_2 v_{12} \right), \quad (\text{II.8})$$

or more explicitly:

$$\begin{aligned} V_\tau(p_1) = & - \frac{2}{(2 \pi \hbar)^3} \frac{2}{\rho_0} \\ & \times \left[T_{F,\tau} \int_{\tau} d^3 p_2 \left(\alpha_l - \beta_l \left(\frac{p_{12}}{p_F} \right)^2 \right. \right. \\ & \left. \left. + \gamma_l \frac{p_F}{|p_{12}|} - \sigma_l \left(\frac{2 \bar{\rho}}{\rho_0} \right)^{2/3} \right) \right. \\ & \left. + T \int_{-\tau} d^3 p_2 \left(\alpha_u - \beta_u \left(\frac{p_{12}}{p_F} \right)^2 \right. \right. \\ & \left. \left. + \gamma_u \frac{p_F}{|p_{12}|} - \sigma_u \left(\frac{2 \bar{\rho}}{\rho_0} \right)^{2/3} \right) \right] \end{aligned} \quad (\text{II.9})$$

Here, $\alpha_l, \beta_l, \gamma_l, \sigma_l$ and $\alpha_u, \beta_u, \gamma_u, \sigma_u$ are defined as: $\alpha_l = 0.5 (1 - \xi) \alpha$; $\beta_l = 0.5 (1 - \zeta) \beta$; $\gamma_l = 0.5 (1 - \zeta) \gamma$; $\sigma_l = 0.5 (1 - \zeta) \sigma$; $\alpha_u = 0.5 (1 - \xi) \alpha$; $\beta_u = 0.5 (1 - \zeta) \beta$; $\gamma_u = 0.5 (1 - \zeta) \gamma$; and $\sigma_u = 0.5 (1 - \zeta) \sigma$.

The results of this model for cold matter are described in more detail in [7–11]. They agree rather well with the values of the semi-empirical droplet mass formula. Interesting is also the equation of state (EOS) for cold neutron star matter, which is characterized by the generalized β -equilibrium between nucleons and leptons, i.e. electrons and muons (for more details, see [16]). One has to obey in this case the additional constraint for the chemical potentials, i.e.

$$\mu_p = \mu_n - \mu_e; \quad \mu_e = \mu_\mu, \quad (\text{II.10})$$

and charge neutrality between the protons and the leptons:

$$\rho_p = \rho_e + \rho_\mu. \quad (\text{II.11})$$

The EOS for this case is contained in Figs. 1 and 2.

III. Thomas-Fermi model at $T > 0$

The generalization of the Thomas-Fermi model for finite temperatures is formally more or less straightforward, since one has mainly to incorporate the Fermi distribution functions into the scheme for zero temperature.

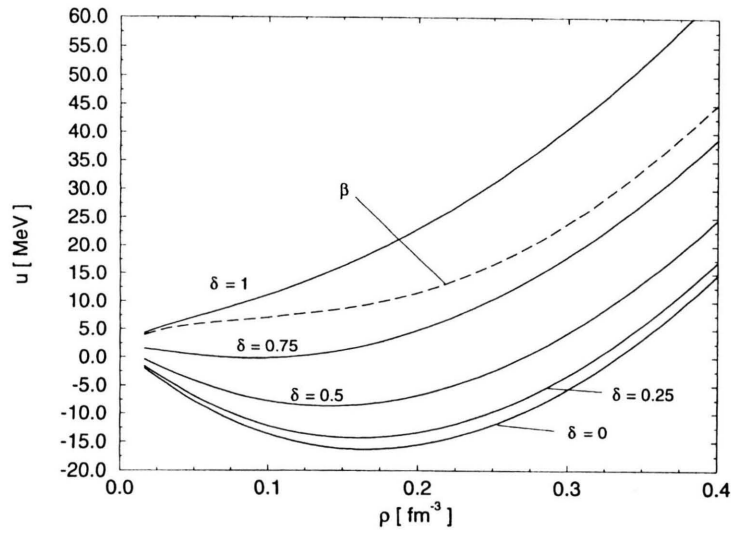


Fig. 1. Energy per nucleon versus density for nuclear matter with different asymmetries and neutron star matter in β -equilibrium. The nuclear matter parameters are: $\rho_0=0.16545 \text{ fm}^{-3}$; $u=16.527 \text{ MeV}$; incompressibility $K=301.27 \text{ MeV}$; asymmetry parameter $J=31.375 \text{ MeV}$. The maximum star mass for a non-rotating neutron star is approximately 2.0 solar masses, and the star radii are between 10 and 12 km.

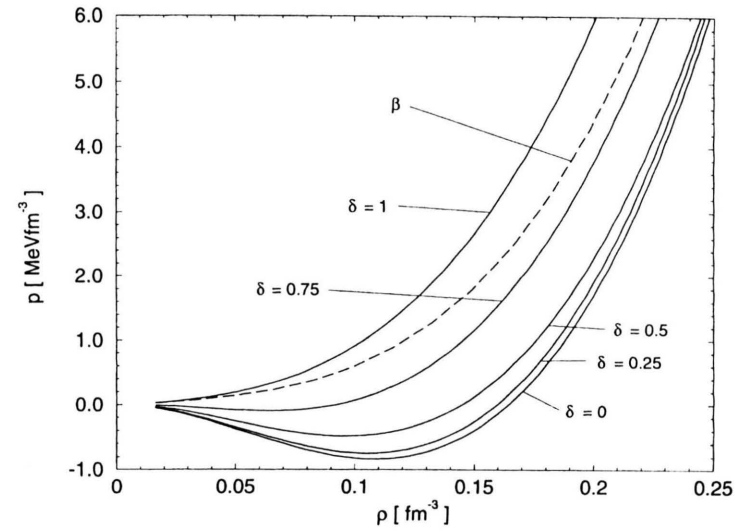


Fig. 2. Pressure as function of density for cold nuclear matter with different asymmetries and neutron star matter.

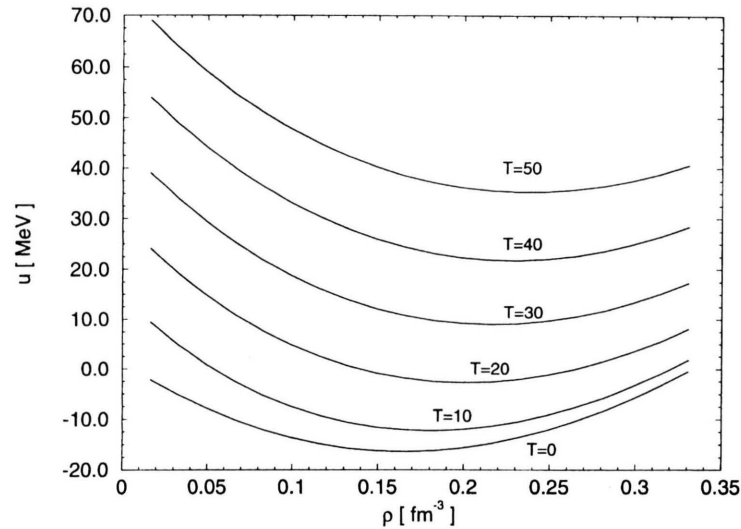


Fig. 3. Energy per nucleon for symmetric nuclear matter ($\delta=0$) versus density at different temperatures (in MeV).

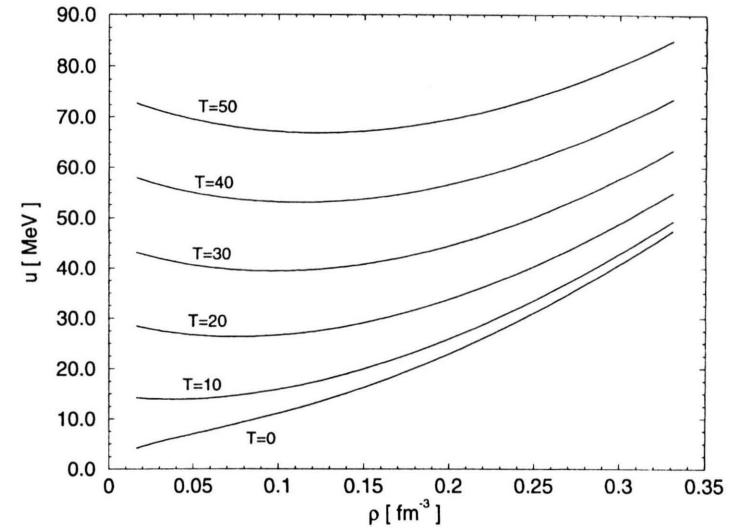


Fig. 4. Energy per nucleon neutron matter ($\delta=1$) versus density at different temperatures (in MeV).

One obtains then for the energy per nucleon

$$u = \frac{2}{\rho (2\pi\hbar)^3} \sum_{\tau} \int_{-\infty}^{+\infty} d^3 p_1 \times \left(\frac{p_1^2}{2m_{\tau}} + \frac{1}{2} V_{\tau}(p_1) \right) f_{\tau}(p_1) \quad (\text{III.12})$$

with the temperature dependent single particle potential

$$V_{\tau}(p_1) = -\frac{2}{(2\pi\hbar)^3} \frac{2T_{F,\tau}}{\rho_0} \times \int_{-\infty}^{+\infty} d^3 p_2 \left(\alpha_l - \beta_l \left(\frac{p_{12}}{p_F} \right)^2 + \gamma_l \frac{p_F}{|p_{12}|} - \sigma_l \left(\frac{2\bar{\rho}}{\rho_0} \right)^{2/3} \right) f_{\tau}(p_2) - \frac{2}{(2\pi\hbar)^3} \frac{2T_F}{\rho_0} \int_{-\infty}^{+\infty} d^3 p_2 \left(\alpha_u - \beta_u \left(\frac{p_{12}}{p_F} \right)^2 + \gamma_u \frac{p_F}{|p_{12}|} - \sigma_u \left(\frac{2\bar{\rho}}{\rho_0} \right)^{2/3} \right) f_{-\tau}(p_2). \quad (\text{III.13})$$

The density is now given by

$$\rho = \frac{2}{(2\pi\hbar)^3} \sum_{\tau} \int_{-\infty}^{+\infty} d^3 p_1 f_{\tau}(p_1). \quad (\text{III.14})$$

The Fermi distribution function, f_{τ} , is defined as ($k_B=1$):

$$f_{\tau}(p_1) = \left(1 + \exp \left(\frac{1}{T} (\varepsilon_{\tau}(p_1) - \mu'_{\tau}) \right) \right)^{-1}, \quad (\text{III.15})$$

where ε_{τ} denotes the single particle energy

$$\varepsilon_{\tau}(p_1) = \frac{p_1^2}{2m_{\tau}} + V_{\tau}(p_1). \quad (\text{III.16})$$

With the single particle energy one can rewrite the energy per nucleon as

$$u = \frac{1}{\pi^2 \hbar^3 \rho} \sum_{\tau} \int_0^{+\infty} d p_1 p_1^2 \frac{1}{2} \times \left(\frac{p_1^2}{2m_{\tau}} + \varepsilon_{\tau}(p_1) \right) f_{\tau}(p_1). \quad (\text{III.17})$$

Due to the momentum dependence of the single particle potential one can also introduce an effective mass via

$$m^* = \left(\frac{1}{p} \frac{\partial \varepsilon(p)}{\partial p} \right)_{p=0}^{-1}. \quad (\text{III.18})$$

For finite temperatures one needs the free energy in order to obtain the pressure and the chemical potentials. The free energy per baryon is given by

$$f = u - T s \quad (\text{III.19})$$

with the entropy per baryon

$$s = \frac{2}{\rho (2\pi\hbar)^3} \sum_{\tau} \int_{-\infty}^{+\infty} d^3 p_1 \cdot (f_{\tau}(p_1) \ln f_{\tau}(p_1) + (1 - f_{\tau}(p_1)) \ln (1 - f_{\tau}(p_1))). \quad (\text{III.20})$$

The expressions for the chemical potentials and the pressure are

$$\mu_{\tau} = \left(\rho \left(\frac{\partial}{\partial \rho_{\tau}} \right)_{\rho-\tau, T} + 1 \right) f \quad (\text{III.21})$$

(μ'_{τ} in (III.15) differs from μ_{τ} , since the interaction is density dependent, see Appendix A of [7]) and

$$P = \rho \sum_{\tau} \rho_{\tau} \left(\frac{\partial f}{\partial \rho_{\tau}} \right)_{\rho-\tau, T}. \quad (\text{III.22})$$

More explicit expressions are given in [11].

IV. Results

First we show in Figs. 1 and 2 for the sake of completeness the EOS for cold nuclear matter with different asymmetries [$\delta := (\rho_n - \rho_p)/\rho$] and neutron star matter. In Figs. 3 and 4 the EOS for symmetric nuclear matter ($\delta=0$) and pure neutron matter ($\delta=1$) is displayed for different temperatures. For small densities, below $\rho=0.0165 \text{ fm}^{-3}$, one obtains, as expected, the behaviour of a free Fermi gas with a linear temperature dependence, since the nucleon-nucleon force has a small range. For increasing density the EOS exhibits a quadratic temperature dependence, since at temperatures below the Fermi temperature a quadratic temperature dependence $u(T) = u(T=0) + a_{\sqrt{T}} T^2$ holds (see [11, 17]). The chemical potential μ , the pressure, the free energy per baryon and the

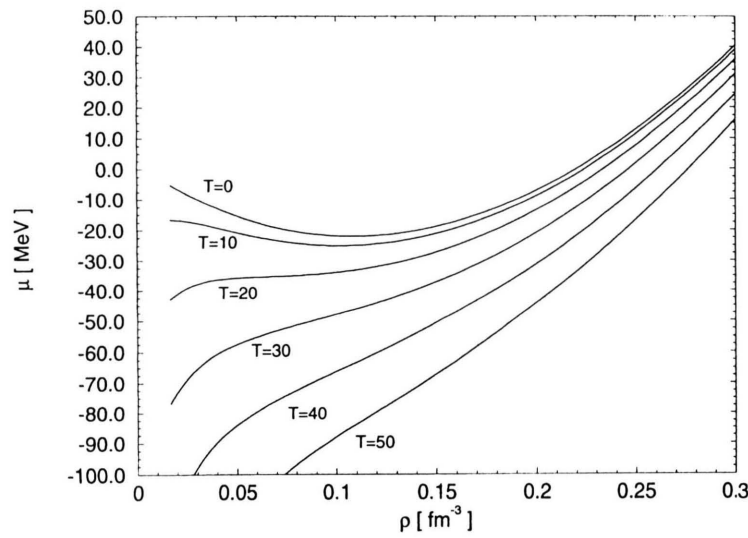


Fig. 5. Chemical potential as function of density for symmetric nuclear matter at different temperatures.

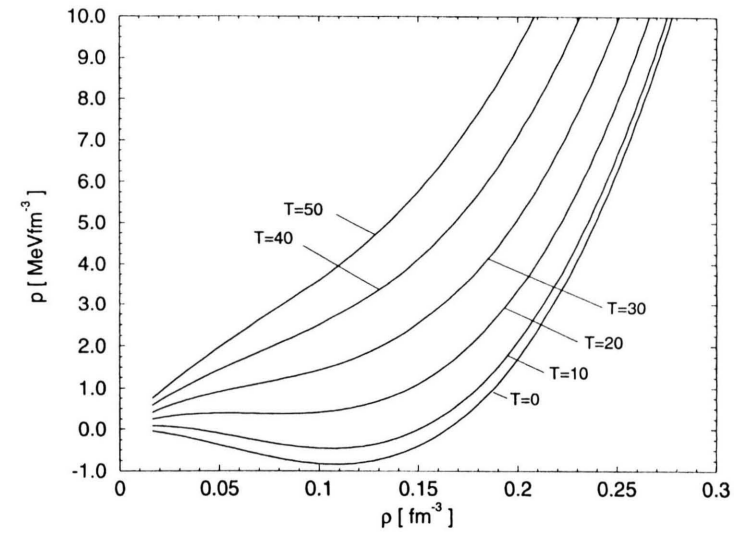


Fig. 6. Pressure versus density for symmetric nuclear matter at different temperatures.

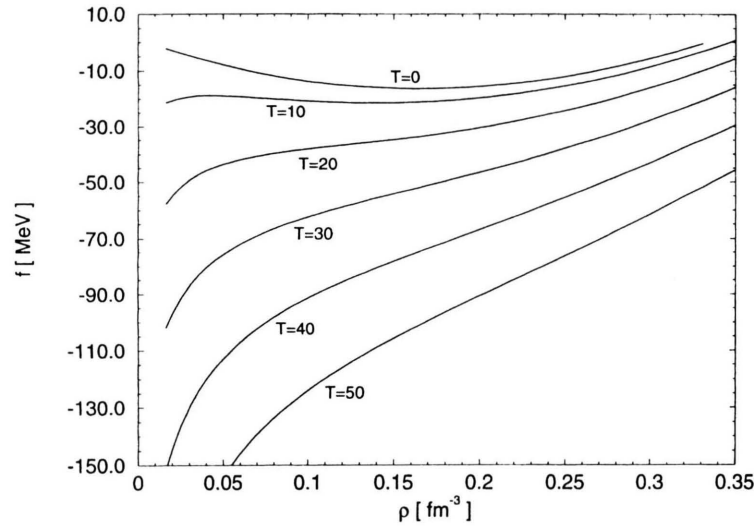


Fig. 7. Free energy per baryon versus density for symmetric nuclear matter at different temperatures.

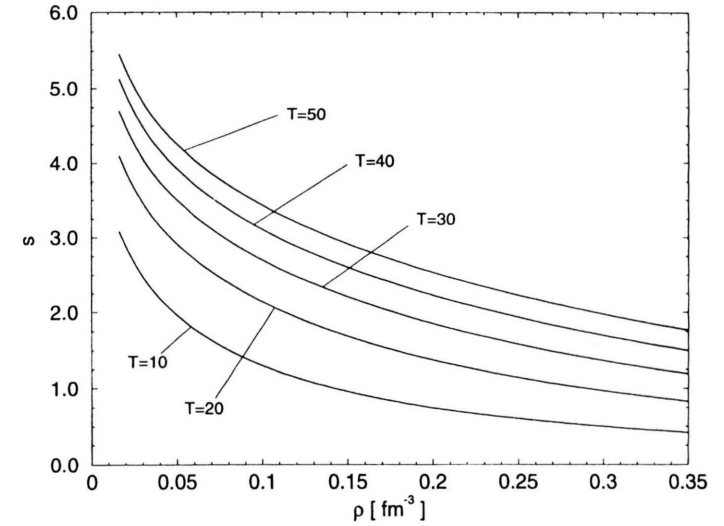


Fig. 8. Entropy per baryon versus density for symmetric nuclear matter at different temperatures. The entropy behaviour agrees with the experimental situation (cf. with Fig. 6 of [6], see also [20]).

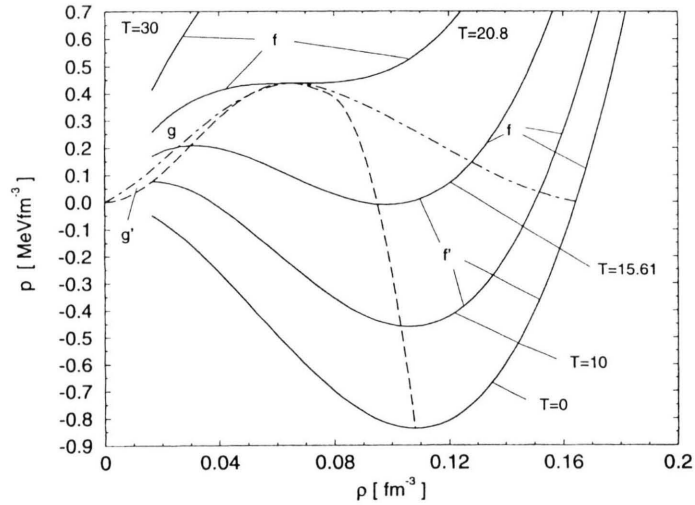


Fig. 9. Liquid-gas phase transition in the P - ρ diagram for symmetric matter: Shown are the isotherms for $T=0, 10, 15.61, 20.8$ and 30 MeV. Above the dashed-dotted curve the matter exists only in one phase, either as gas (g) between the dashed-dotted line and the isotherm with the critical temperature $T_c=20.8$ or as liquid (f) above $T_c=20.8$ MeV. Between the dashed-dotted curve and the dashed curve one finds metastable phases, again either as a gas (g') or a liquid (f'). Below the dashed curve the state is not stable. Shown is also the pressure at the flash temperature (15.61 MeV).

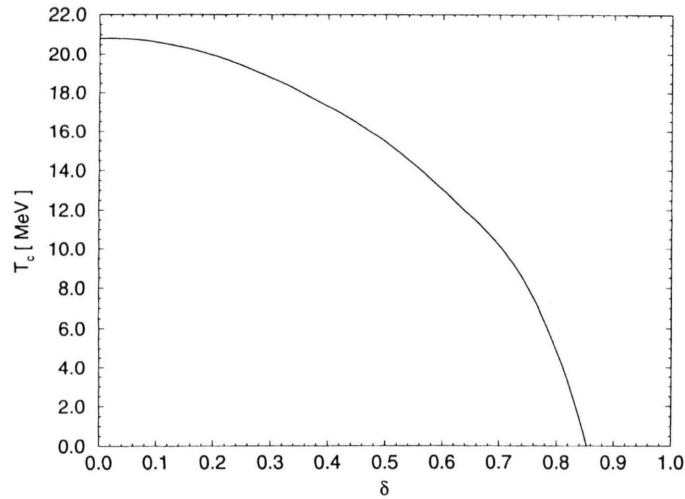


Fig. 11. Asymmetry dependence of the critical temperature T_c .

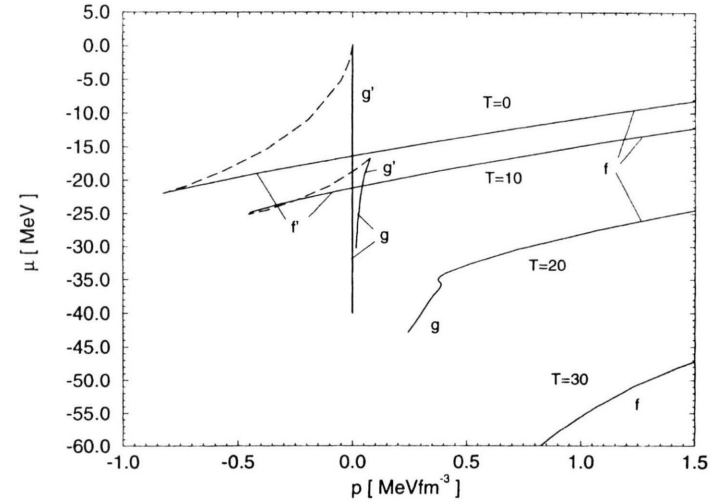


Fig. 10. Chemical potential as function of the pressure for symmetric nuclear matter. The crossing point on each isotherm is the condition of phase equilibrium.

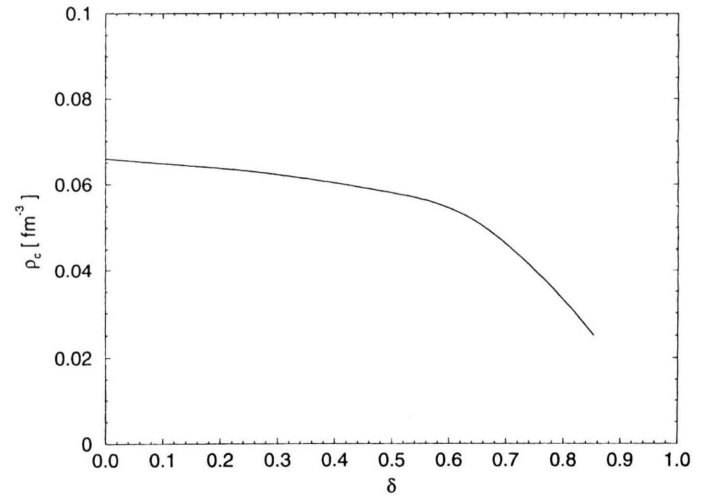


Fig. 12. Asymmetry dependence of the critical density ρ_c .

entropy per baryon for symmetric matter are exhibited in Figs. 5, 6, 7 and 8 for different temperatures. The values of the entropy per baryon are in good agreement with experimental results (see [20]). This is important, since the entropy per baryon is also a necessary ingredient for the calculation of protoneutron stars (see e.g. [18, 19]).

Due to the non-monotonic behaviour of the isotherms one can infer that these systems undergo a liquid-gas phase transition. Such a transition, which may occur in warm matter produced in heavy-ion collisions, is one of the most interesting problems in modern nuclear physics. Several groups are investigating this problem by studying multifragmentation, where the coexistence of nuclear fragments and free nucleons could be considered as a signal for the transition phase (see, for instance, [21, 22]). The critical temperature can be obtained by a Maxwell construction. The situation can be either viewed in the P - ρ -diagram or in the μ - P -diagram. These views are given for symmetric nuclear matter in Figs. 9 and 10, respectively. The Gibbs condition of phase equilibrium demands that the pressure, temperature and chemical potential are equal in the two phases.

As can be seen from the EOSs, the incompressibility of nuclear matter decreases with increasing temperature. As a consequence one should expect a limiting temperature, at which a self-bound system can still exist in hydrostatic equilibrium ($P=0$). Beyond this temperature, the so-called flash temperature T_f , the unbound system starts expanding [23]. The isothermal incompressibility is defined as

$$K_T = 9 \left(\rho^2 \frac{\partial^2 f}{\partial \rho^2} + 2 \rho \frac{\partial f}{\partial \rho} \right)_T, \quad (\text{IV.23})$$

and is zero at the point where the minimum pressure reaches the zero pressure line (here at a density of $\rho = 0.0973 \text{ fm}^{-3}$). We obtain for the flash temperature the value 15.61 MeV which is, as expected, smaller than the

critical temperature T_c because at T_c the pressure is already positive (see Fig. 6). If one increases the asymmetry of the system, the critical temperature and the critical density decrease. These dependencies are depicted in Figs. 11 and 12. For asymmetries larger than 0.853 the liquid-gas phase coexistence disappears in our model.

V. Conclusions

We have calculated the EOS of cold and warm nuclear matter for different asymmetries using the approach of Myers and Świątecki. Almost all non-relativistic calculations give critical temperatures in the range of 14–22 MeV and critical densities of approximately 1/3 of the saturation density [4], which is in accordance with our results. Investigations for finite temperatures are relatively scarce, and the model seems to provide a simple theoretical tool for investigating the nuclear EOS in such ranges, which seems to be of considerable interest, since it becomes now possible by means of heavy-ion physics to study the behaviour of matter off the saturation point, which is necessary in the theory of neutron stars and supernova explosions and also in the interpretation of heavy-ion collisions [24].

It seems worthwhile, due to its simplicity and excellent agreement with nuclear data, to use this model with possible improvements in further non-relativistic investigations of the EOS. Also the properties of neutron stars seem to be in accordance with this approach [16].

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